THERMAL CONDUCTIVITY OF GLASS-FIBER SYSTEMS

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Heat transfer in glass-fiber materials is considered. The effect of scattering on heat transfer is investigated, and the results of calculations are compared with published data.

According to the terminology developed in [1, 2], fiber systems are dispersed media with communicating pores. Dul'nev et al. [2, 3] have proposed a model with long-range order in the form of mutually penetrating rods to describe such systems with randomly distributed fibers (cotton wool, felt, and so on). The following expression for the effective thermal conductivity is obtained:

$$\lambda_{\text{eff}} = \lambda_1 \left[c^2 + \nu (1-c)^2 + \frac{2\nu c (1-c)}{\nu c + (1-c)} \right], \quad \nu = \frac{\lambda_2}{\lambda_1}. \tag{1}$$

The parameter c is the solution of the equation

$$p = 2c^3 - 3c^2 + 1,$$

where p is the porosity of the system [$p = V_2/(V_1 + V_2)$]. It is shown in [4] that this model accounts quite satisfactorily for the measured effective thermal conductivity of dispersed systems in a broad range of values of p.

The particular feature of glass-fiber systems used in industry is their high porosity (0.80–0.99) and elongation (ratio of fiber length to diameter in excess of 1000). Such systems can be described by a simpler model than that considered above.

Suppose that a unit volume contains n fibers of unit length and radius r, and let τ and $(1 - \tau)$ be the fraction of fibers elongated at right-angles to and in the direction of the heat flow, respectively.

Let us suppose that τ_n fibers and the gas filler in a unit volume constitute the thermal resistances R_1' and R_2 connected in series, while $(1 - \tau)n$ fibers form the resistance R_1'' connected in parallel to them. We then have

$$R_1' = \frac{l_1}{\lambda_1 S_2}, \quad R_1'' = \frac{1}{\lambda_1 S_1}, \quad R_2 = \frac{l_2}{\lambda_2 S_2}. \tag{2}$$

In this expression $S_1 = \pi r^2 n(1 - \tau)$ is the cross-sectional area of all the fibers which are parallel to the heat flow; $S_2 = 1 - \pi r^2 n(1 - \tau)$ is the total area corresponding to the gas filler and the fibers which lie at right-angles to the heat flow; $l_1 = \tau \pi r^2 n/S_2$, $l_2 = V_2/S_2$ are the equivalent dimensions corresponding to the τn fibers and the gas filler in unit volume, respectively.

The total thermal resistance per unit volume of the system $R_{\text{eff}} = 1/\lambda_{\text{eff}}$ is given by

$$\frac{1}{R_{\text{eff}}} = \frac{1}{R_1''} + \frac{1}{R_1' + R_2}. \tag{3}$$

Substituting for R_1', R_1'', and R_2 from Eq. (2) into Eq. (3), and using the formula for the porosity per unit volume

$$p = 1 - \pi r^2 n, \tag{4}$$

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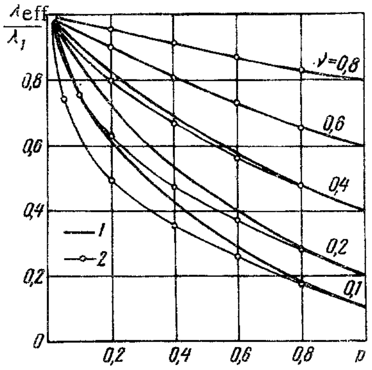

Fig. 1. λ_{eff}/λ_1 for a system with communicating pores as a function of p and ν: 1) according to Eq. (1); 2) according to Eq. (5').

we have (for conductive heat transfer)

$$\lambda_{eff} = \lambda_1 \left\{(1-p)(1-\tau) + \frac{[1-(1-p)(1-\tau)]^2}{\tau(1-p) + \frac{p}{\nu}}\right\}. \quad (5)$$

When the fiber directions are uniformly distributed [a similar assumption is adopted in the derivation of Eq. (1)], i.e., when $\tau = 2/3$, we have

$$\lambda_{eff} = \lambda_1 \left[\frac{1-p}{3} + \frac{(2+p)^2}{6(1-p) + \frac{9p}{\nu}}\right]. \quad (5')$$

In Fig 1 values of λ_{eff}/λ_1 calculated from Dul'nev's formula (1) and from Eq. (5'), for different p and ν, are compared. It is clear from Fig. 1 that the two sets of calculated values of λ_{eff}/λ_1 are practically the same for $p > 0.80$. For $p > 0.80$ and $\nu < 0.07$ (for example, for glass wool in air under normal conditions $\nu = 0.027$) we can simplify Eq. (5') as follows:

$$\lambda_{eff} = \lambda_1 \left[\frac{1-p}{3} + \frac{(2+p)^2}{9p}\nu\right]. \quad (6)$$

The difference between Eqs. (5') and (6) is then not more than 1%.

In addition to conductive heat transfer through the fibrous system, we can also have convective, radiative, and contact transfer. It follows from [1, 5, 6] that in the glass-fiber systems which we are considering we can neglect convective and contact heat transfers. Radiative heat transfer can be taken into account through the simultaneous solution of two integrodifferential equations, one of which describes the radiative heat transfer and the other the conservation of energy. The solution of this set of equations is mathematically very difficult, and in the published material there are only numerical or very unwieldy solutions. The problem is substantially simplified by using the differential-difference approximation of Schuster and Schwarzschild. For a purely scattering medium the solution of the differential-difference equation yields the following expression for the radiation flux through a plane-parallel layer (see, for example, [7]):

$$q_r = \int_0^\infty \frac{E_\lambda(T_1) - E_\lambda(T_2)}{\varepsilon + \beta_\lambda L}\, d\lambda. \quad (7)$$

In this expression $\beta_\lambda = kd\tau n$ is the scattering coefficient of the medium; and it is assumed that the radiation is attenuated by the fibers which lie in planes parallel to the bounding surfaces; $\varepsilon = (1/\varepsilon_1) + (1/\varepsilon_2) - 1$.

It is shown in [8] that when the radiation is incident at right-angles to the axis of an infinitely long cylinder, the scattering coefficient of the cylinder is very close to that for a sphere (for refractive indices $m < 2.5$):

$$k = 2 - 4\rho^{-1}\sin\rho + 4\rho^{-2}(1-\cos\rho), \quad (8)$$

where $\rho = 2\pi d(m-1)/\lambda$.

If $E_\lambda(T)$ takes the form of Planck's law, and we use Eqs. (4) and (8), we can rewrite Eq. (7) in the form

$$q_r = \int_0^\infty \frac{C_1\lambda^{-5}\left\{\left[\exp\left(\frac{C_2}{\lambda T_1}\right)-1\right]^{-1}-\left[\exp\left(\frac{C_2}{\lambda T_2}\right)-1\right]^{-1}\right\}d\lambda}{\varepsilon+\frac{4\tau(1-p)L}{\pi d}\left[2-4\rho^{-1}\sin\rho+4\rho^{-2}(1-\cos\rho)\right]}. \tag{9}$$

In this expression $C_1 = 0.374\cdot 10^{-15}$ W·m², $C_2 = 0.0144$ m·deg. To obtain an analytic expression for the integrals in Eq. (9), we replace k by the approximate formula

$$k = \frac{2\rho^2}{1+\rho^2}. \tag{10}$$

Assuming that the refractive index of the glass fibers is constant and equal to 1.5 (which is fully justified provided the temperatures are not too high), and evaluating the integral in Eq. (9) with k given by Eq. (10), we obtain

$$q_r = \frac{C_1}{(\pi d)^4\varepsilon A^6}\left\{\frac{D}{15}(T_1^4-T_2^4)+\frac{(A^2-1)D}{6}(T_1^2-T_2^2)\right.$$
$$\left.+\frac{1-A^2}{2}\left[\ln\left(\frac{T_2}{T_1}\right)-D(T_1-T_2)+\Psi((2DT_1)^{-1})-\Psi((2DT_2)^{-1})\right]\right\}, \tag{11}$$

where $A^2 = 1+(8\tau(1-p)L)/\pi d\varepsilon$; $D = \pi^2 dA/C_2$; Ψ is the Euler function. In obtaining this solution we made use of the results given in [9].

Numerical estimates show that the term in the square brackets in Eq. (11) can be neglected in comparison with the first two terms, so that we have

$$q_r = a\left[(T_1^4-T_2^4)+b(T_1^2-T_2^2)\right], \tag{12}$$

where

$$a = \frac{\sigma}{\varepsilon+\frac{8\tau(1-p)L}{\pi d}}; \quad b = \frac{20C_2^2\tau(1-p)L}{\pi^5 d^3\left(\varepsilon+\frac{8\tau(1-p)L}{\pi d}\right)};$$

$\sigma = 5.67\cdot 10^{-8}$ W/m²·deg⁴ is the Stefan–Boltzmann constant.

Table 1 gives the measured [10] and calculated [from Eq. (12)] heat fluxes in vacuo between surfaces at temperatures of 300 and 77°K separated by glass-fiber insulation. It was assumed that $\tau = 1$ since SBR-M glass paper and EVTI-15 glass cloth contain fibers which lay mainly at right-angles to the direction of the heat flux.

In layered thermal insulation in vacuum the fraction of radiative flux is shown by Eq. (12) to be higher by 10-30% than the values reported in [10, 11, 12] and based on the usual formula, which does not take into account attenuation of radiation by scattering. The remaining results and conclusions (the explanation of the "thermal paradox" and the dependence of thermal conduction on the thickness of the layered insulation in vacuo due to the residual gases) are as before. At higher temperatures the discrepancy between calculations based on Eq. (2) and the experimental data is greater (cf. [13]). It is clear that a more accurate analysis of radiative transfer is required in this case.

Equation (12) can be used to define in the usual way the radiative heat-transfer coefficient:

$$q_r = \lambda_r(T_1-T_2)/L. \tag{13}$$

When $8\tau(1-p)L/\pi d \gg \varepsilon$ we have

$$\lambda_r = \frac{\pi\sigma}{8\tau(1-p)}\left[(T_1^2+T_2^2)(T_1+T_2)d+\frac{5C_2^2}{2\pi^4 d}(T_1+T_2)\right]. \tag{14}$$

For small temperature gradients

$$\lambda_r = \frac{\pi\sigma}{8\tau(1-p)}\left(4T^3 d+\frac{5C_2^2}{\pi^4 d}T\right). \tag{15}$$

Figure 2 shows experimental data [6] with values of λ_r in vacuum and the values obtained from Eqs. (14), (15), and (9). The integral in Eq. (9) is evaluated by Simpson's rule. It is clear from Fig. 2a that Eq. (9) is satisfactorily approximated by either Eq. (12) or Eqs. (14) and (15). We used $\tau = 2/3$ in the calculations. In the worst case, calculation and experiment differ by a factor of 2-2.5. On the other hand, the discrepancy in the case of the formulas proposed in [3, 6] is an order of magnitude.

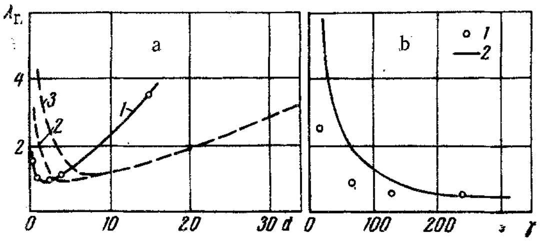

Fig. 2

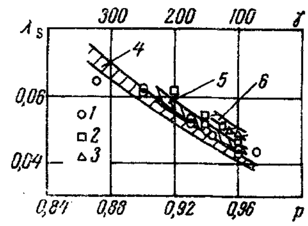

Fig. 3

Fig. 2. The quantity λ_r, mW/m·deg, for glass wool in vacuum: a) as a function of glass-fiber diameter d, μ; mean temperature 297°K; 1) experiment [6]; 2) Eq. (15); 3) Eq. (9); b) as a function of the density γ, kg/m^3; d = 1.15 μ surface temperatures 293 and 90°K; 1) experiment [6]; 2) Eq. (14).

Fig. 3. Dependence of λ_s, W/m·deg, for glass wool in air on the porosity p and density γ, kg/m^3 (mean temperature 303°K); 1) experiment [14-16], d = 5 μ; 2) experiment [14, 15], d = 35 μ; 3) experiment [15], d = 104 μ; 4-6) calculation based on Eq. (17) with d = 5.35 and 104 μ, respectively.

TABLE 1. Heat Fluxes in Vacuo between Surfaces at Temperatures of 300 and 77°K

Insulator	q_r, W/m^2 expt. [10]	q_r, W/m^2 Eq. (12)
SBR-M glass paper	4,44	4,3
EVTI-15 Glass cloth	10,4	8,4

Equation (14) can be used to determine the optimum glass-fiber diameter, i.e., the diameter for which λ_r is a minimum. Differentiating Eq. (14) with respect to d and equating to zero, we obtain

$$d = \frac{\sqrt{5}\, C_2}{2\pi^2 \sqrt{T_1^2 + T_2^2}}. \tag{16}$$

It is clear from this expression that for thermal insulation in vacuo with reduced boundary temperatures it is more convenient to use thicker glass fibers (which are also less expensive).

The resultant heat-transfer coefficient for the glass-fiber system is

$$\lambda_s = \lambda_{eff} + \lambda_r, \tag{17}$$

where λ_{eff} is determined by Eq. (5) or Eq. (5') and λ_r by Eq. (14), (15), or (9).

That the resultant heat-transfer coefficient can be written as the sum of conductive and radiative components is a consequence of the Schuster–Schwarzschild approximation.

The experimental data in [14-16] and those calculated from Eq. (17) are given in Fig. 3. The thermal conductivity of glass fibers is taken from the experimental data of L.S. Eigenson, S.A. Serdobol'skaya et al. (Scientific-Research Institute for Glass, Moscow), who determined the thermal conductivity of glass with a composition close to that of the glass fiber. The shaded regions in Fig. 3 correspond to the thermal conductivity of glass fiber for λ_1 = 0.95-1.05 W/m·deg. The contribution of the radiative component λ_r to the resultant heat-transfer coefficient of glass wool for fibers with d = 5 μ is 0.5-3.0% in the above porosity range and 2.0-7.5% and 14% for d = 35 μ and d = 104 μ, respectively.

Estimates of the convective component for glass-fiber insulation using the formulas given in [1] under normal industrial conditions (temperature drop 50°K, thickness 0.05 m) show that convection appears when d > 150-300 μ for a porosity p = 0.96-0.92.

It is clear from the foregoing that the contribution of the radiative component to the thermal conductivity of glass-fiber systems is quite small. Since the cost of glass-fiber components with fiber diameters of 35 μ is several times the cost in the case of diameters of 100 μ, it is better to use components made of glass fibers of diameter 100 μ as a heat-insulating material especially in housing and industrial buildings. When p = 0.92, one can obviously use glass fibers with diameters up to 300 μ for these purposes. It is shown in [17, 18] that constructional materials consisting of elastic (35-100 μ) and thick (100-300 μ) fibers are stronger and have a longer life that components consisting of fine (3-12 μ) and coarse (12-30 μ) glass fibers. The sound insulating properties remain roughly the same as for fine and coarse glass fibers.

NOTATION

λ_c	is the thermal conductivity of fiber system;
λ_{eff}, λ_r	are the conduction and radiation thermal conductivity;
λ_1, λ_2	are the fiber and gas-filler thermal conductivity;
V_1, V_2	are the volumes of fibers and gas-filler using unit volume of system;
n	is the number of fibers per unit volume;
r, d	are the radius and diameter of glass fiber;
R	is the thermal resistance;
S	is the area;
l	is the length;
$E_\lambda(T_1)$, $E_\lambda(T_2)$	are the spectral intensities of thermal radiation density for boundary surfaces;
T_1, T_2	are the boundary surface temperatures;
ε_1, ε_2	are the emissivities of boundary surfaces;
L	is the distance between boundary surfaces;
k	is the dissipation factor of medium particles;
λ	is the wavelength;
m	is the refraction index;
γ	is the density.

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